

Math 116 Section 04

Midterm 2

Name _____

August 08, 2005

Instructor: Charles Cuell

Student Number _____

All solutions are to be presented on the paper in the space provided. The exam is closed book, no calculators. Time for the exam is 50 minutes.

(1) (5 marks) Find the area between the curves $y = (x-1)^2$, $y = 0$ and $x = 0$.

$$\begin{aligned} A &= \int_0^1 (x-1)^2 dx \\ &= \frac{1}{3}(x-1)^3 \Big|_0^1 \\ &= \frac{1}{3}((1-1)^3 - (0-1)) \\ &= \frac{1}{3} \end{aligned}$$

(2) (5 marks) Find the volume obtained by rotating the region bounded by $y = \sin x, y = 0, x = 0, x = \pi$ about the x -axis.

By disks:

$$\begin{aligned} V &= A(x) dx \\ &= \int_0^\pi \pi r^2 dx \\ &= \pi \int_0^\pi \sin^2 x dx \\ &= \pi \int_0^\pi \frac{1}{2}(1 - \cos(2x)) dx \\ &= \frac{\pi}{2} \left(x - \frac{1}{2} \sin(2x) \right) \Big|_0^\pi \\ &= \frac{\pi}{2} \left(\pi - \frac{1}{2} \sin(2\pi) - (0 - \frac{1}{2} \sin 0) \right) \\ &= \frac{\pi^2}{2} \end{aligned}$$

(3) (5 marks) A spring has natural length 0.02m and spring constant 10 kg s^{-2} . How much work is required to stretch the spring to a length of 0.03m?

$$\begin{aligned} W &= \int_a^b F(x) dx \\ &= \int_{0.02}^{0.03} k(x - x_0) dx \\ &= \int_{0.02}^{0.03} 10(x - 0.02) dx \\ &= 10 \frac{(x - 0.02)^2}{2} \bigg|_{0.02}^{0.03} \\ &= 10 \left(\frac{(0.03 - 0.02)^2}{2} - \frac{(0.02 - 0.02)^2}{2} \right) \\ &= 0.0005 \text{ N} \end{aligned}$$

(4) (5 marks) Find the average value of $f(x) = \frac{1}{x}$ over the interval $[\frac{1}{2}, 2]$

$$\begin{aligned}
 f_{\text{ave}} &= \frac{1}{b-a} \int_a^b f(x) dx \\
 &= \frac{1}{2 - \frac{1}{2}} \int_{\frac{1}{2}}^2 \frac{1}{x} dx \\
 &= \frac{2}{3} \ln x \Big|_{\frac{1}{2}}^2 \\
 &= \frac{2}{3} \left(\ln 2 - \ln \frac{1}{2} \right) \\
 &= \frac{2}{3} (\ln 2 + \ln 2) \\
 &= \frac{4}{3} \ln 2
 \end{aligned}$$